The Radiosity Method in Optical Remote Sensing of Structured 3-D Surfaces

BY

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Monochromatic intensity : $I_\nu(\mu, \phi)$

$$
\begin{aligned}
I_\nu &= \frac{Energy}{Area \ \times \ Time \ \times \ Stearadian \ \times \ (1 \ / \ Time)} \\
&= \frac{Energy}{Area \ \times \ Stearadian}
\end{aligned}
$$

Reflectance : $\rho_\nu(\mu, \phi, \mu', \phi')$

$$
I_\nu(\mu, \phi)_{reflected} = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 I_\nu(\mu', \phi') \ \rho_\nu(\mu, \phi, \mu', \phi') \ \mu' \ d\mu' \ d\phi
$$

Emittance : $\varepsilon_\nu(\mu, \phi)$

$$
\begin{aligned}
I_\nu(\mu, \phi)_{em} &= \left(1 - \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \rho_\nu(\mu, \phi, \mu', \phi') \mu' d\mu' d\phi'\right) I_{b,\nu}(T) \\
&= \varepsilon_\nu(\mu, \phi) \ I_{b,\nu}(T)
\end{aligned}
$$

Planck's black body radiation : $I_{b,\nu}(T)$

$$
I_{b,\nu}(T) = \frac{2h\nu}{c^2} \frac{1}{h\nu \exp(-\frac{h\nu}{kT}) - 1}
$$

GENERAL RADIOSITY EQUATION FOR AN ENCLOSURE

•Equilibrium condition :

$$I_\nu(\mu,\phi)_{leaving} \;=\; I_\nu(\mu,\phi)_{emitted} \;+\; I_\nu(\mu,\phi)_{reflected}$$

•As a function of position (x,y) **:**

$$\begin{aligned} I_\nu(x,y,\mu,\phi) \quad = \quad & \varepsilon_\nu(x,y,\mu,\phi)\; I_{b,\nu}[T(x,y)] \\ & + \frac{1}{\pi}\int_0^{2\pi}\int_0^1 I_\nu(x,y,\mu',\phi') \\ & \quad \rho_\nu(x,y,\mu,\phi,\mu',\phi')\;\mu'\;d\mu'\;d\phi \end{aligned}$$

•Solid angle of surface $d\xi\;d\eta$ **at** (x,y) **:**

$$d\mu'\;d\phi' \;=\; \frac{\zeta\;d\xi\;d\eta}{[r(x,y,\xi,\eta)]^2}$$

- **Leaving intensity at (x, y) as a function of the leaving intensity at (ξ, η) :**

$$
\begin{aligned}
I_\nu(x, y, \mu, \phi) &= \varepsilon_\nu(x, y, \mu, \phi) I_{b,\nu}[T(x, y)] \\
&\quad + \frac{1}{\pi} \int_{Enc} \frac{I_\nu(\xi, \eta, \zeta, \psi) \rho_\nu(x, y, \mu, \phi, \mu', \phi')}{[r(x, y, \xi, \eta)]^2} \\
&\qquad \mu' \ \zeta \ d\xi' \ d\eta
\end{aligned}
$$

- **For diffuse surface :**

$$
\rho_\nu(x, y, \mu, \phi, \mu', \phi') = \rho_\nu(x, y)
$$

- **Radiosity for a diffuse surface :**

$$
B_\nu(x, y) = \pi \ I_\nu(x, y)
$$

- **Radiosity Integral Equation:**

$$
\begin{aligned}
B_\nu(x, y) &= \pi \ \varepsilon_\nu(x, y) \ I_{b,\nu}[T(x, y)] \\
&\quad + \frac{\rho_\nu(x, y)}{\pi} \int \int_{Enc} B_\nu(\xi, \eta) \frac{\mu' d\xi d\eta}{[r(x, y, \xi, \eta)]^2}
\end{aligned}
$$

•Lumped System Approximation :

$$B_{\nu,i} = \pi \ \varepsilon_{\nu,i} \ I_{b,\nu}(T_i) + \frac{\rho_{\nu,i}}{\pi} \sum_{j=1}^{N} B_{\nu,j} \int_{S_j} \frac{\mu_i \ \mu_j}{r_{ij}^2} dS_j,$$
$$i = 1, 2, \ldots, N$$

• Integrate over S_i and divide by S_i :

$$B_{\nu,i} = \pi \ \varepsilon_{\nu,i} \ I_{b,\nu}(T_i) + \rho_{\nu,i} \sum_{j=1}^{N} B_{\nu,j} \ F_{ij}$$

where F_{ij} is called form factor or view factor and is defined as :

$$F_{ij} = \frac{1}{S_i} \int_{S_i} \int_{S_j} \frac{\mu_i \ \mu_j}{\pi \ r_{ij}^2} dS_j \ dS_i$$